Nonlocal field theory, rolling tachyons, and initial value problem

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— to appear next week!

Organization of the talk!

1. Introduction to the issues and model

2. Redefining the time-dependent theory

3. Redefinitions of the general nonlocal theory

4. Rolling tachyons

Introduction to the issues and model

— String field theories display nonlocalities.

— In Lorentz-covariant formulations these are nonlocalities in time and in space.

— The nonlocalities ensure the convergence properties of string amplitudes and the absence of UV divergences (in Euclidean space).

— But time-nonlocalities are considered problematic.

— Without an initial value problem we cannot predict future evolution.

— There could also be violations of causality and unitarity.

1. Are there reformulations of a non-local theory that make it clear that there is an initial value problem?

2. For Lorentz-covariant string field theory we have light-cone string field theory. This action only has a first-order time derivative $\partial/\partial x^+$ in the kinetic term!

3. The relation between covariant and light-cone SFT has largely been established by Erler and Matsunaga [arXiv:2012.09521].

4. Eliezer and Woodard (1989): Light-cone SFT filters out non-sensical solutions of covariant SFT. The theories are only equivalent in perturbation theory.

OSFT has rolling tachyon solutions that seem very puzzling.

The tachyon rolls past the ‘tachyon vacuum’ and then begins oscillations of ever-increasing amplitude.

There is no evidence, within the non-local field theory that the final state is the zero-pressure state of ‘tachyon matter’. But, there is good evidence within SFT that the final state **is** tachyon matter.

Thus there are a few options:

- The rolling solution is tachyon matter, and a proper analysis of the nonlocal field theory is not available yet.
- In the rolling solution the contribution of open string massive states is crucial to get the right answer.
- The rolling solution is not present in the local reformulation of the nonlocal theory.

Open string tachyon field: $p^2 = -m^2 = \frac{1}{\alpha'}$.

$$\eta_{\mu\nu} = \text{diag}(-1, 1, \ldots, 1)\,, \quad \partial^2 = -\partial_t^2 + \nabla^2\,.$$

OSFT Lagrangian truncated to the (unit-free) tachyon field ϕ:

$$L \sim \tfrac{1}{2}\phi\big(\alpha'\partial^2 + 1\big)\phi + \tfrac{1}{3}\big(e^{\log\gamma(\alpha'\partial^2+1)}\phi\big)^3\,, \qquad \gamma = \tfrac{3\sqrt{3}}{4}\,.$$

Use α' to scale the derivatives,: $\sqrt{\alpha'}\partial \to \partial$:

$$L \sim \tfrac{1}{2}\phi\big(\partial^2 + 1\big)\phi + \tfrac{1}{3}\big(e^{\xi^2(\partial^2+1)}\phi\big)^3\,, \qquad \xi^2 = \log\gamma = 0.26162\,.$$

ξ is the nonlocality parameter.

$\xi = mL_\xi\,,$ with L_ξ the nonlocality scale.

The general tachyon model:

$$L \sim \tfrac{1}{2}\phi\big(\partial^2 + m^2\big)\phi + \tfrac{1}{3}g\Big(e^{a^2(\partial^2+m^2)}\phi\Big)^3 .$$

After scaling coordinates, and fields, this can be put in the form:

$$\boxed{L \;=\; \tfrac{1}{2}\,\phi\big(\partial^2 + 1\big)\phi + \tfrac{1}{3}\big(e^{\xi^2\partial^2}\phi\big)^3 .}$$

- Treat ξ as a parameter $\rightarrow$ better analytic control.
- Allows one to explore small and large nonlocality.
- A ξ^2 expansion is a derivative expansion— thus we work in the framework of **effective field theory**.
- For p-adic strings we have $\xi^2(p) = \frac{1}{4}\log p$. $\big[\xi^2(2) = 0.1733\big]$

$$L = \tfrac{1}{2}\,\phi\big(\partial^2 + 1\big)\phi + \tfrac{1}{3}\big(e^{\xi^2\partial^2}\phi\big)^3 .$$

Conventional wisdom:

One can eliminate the nonlocality in perturbation theory.

The reality is a lot more nuanced, subtle, and interesting. This is the subject of this talk.

As in effective field theory, or more generally, for theories without spacetime topology, we <u>ignore total derivatives</u> in L

Redefining the time-dependent theory

A *higher derivative* means two or more derivatives act on a single field.

A higher-derivative *term* is one in which there is at least one higher derivative, after trying to eliminate it by making use of integration by parts.

$\partial^2\phi$ is a higher derivative

$\phi\partial^2\phi \simeq -\partial\phi\partial\phi$ is *not* a higher-derivative term

$\phi\partial^2\phi\partial^2\phi$ is a higher-derivative term

We now focus on the theory when fields only have time dependence:

$$L \;=\; -\tfrac{1}{2}\,\phi\Big(\partial_t^2 - 1\Big)\phi + \tfrac{1}{3}\Big(e^{-\xi^2\partial_t^2}\phi\Big)^3 .$$

Claim 1: All higher-derivative terms in L can be removed by field redefinitions in an expansion in ξ^2.

Claim 2: All terms of the form $\phi^k(\dot\phi)^n$ with integers $n > 2$ and $k \geq 0$, can also be eliminated. (Note that $\phi^k\dot\phi \sim 0$)

$\rightarrow$ The redefined L has a canonical kinetic term and a ξ^2 dependent potential.

Sketch of a proof. Expand the Lagrangian

$$L = L_0 + \xi^2 L_2 + \mathcal{O}(\xi^4)\,,$$

$$L_0 \;=\; -\tfrac{1}{2}\,\phi\Big(\partial_t^2 - 1\Big)\phi + \tfrac{1}{3}\phi^3\,, \quad V(\phi) = -\tfrac{1}{2}\phi^2 - \tfrac{1}{3}\phi^3\,.$$

$$L_2 = -\phi^2 \partial_t^2 \phi$$

Now do a field redefinition $\phi \to \phi + \delta\phi$ with

$$\delta\phi = \xi^2 \delta_1 \phi + \mathcal{O}(\xi^4)\,,$$

We find:

$$L[\phi + \delta\phi] = \; L_0 - \xi^2 (\delta_1\phi)\,(\partial_t^2\phi + V'(\phi)) - \xi^2\phi^2\partial_t^2\phi + \mathcal{O}(\xi^4)\,.$$

Choose $\delta_1\phi = -\phi^2$ to cancel the higher time derivatives:

$$\begin{aligned} L[\phi + \delta\phi] = &\; L_0 + \xi^2\phi^2\, V'(\phi) + \mathcal{O}(\xi^4) \\ = &\; -\tfrac{1}{2}\,\phi\partial_t^2\phi - \tfrac{1}{2}\phi^2 + \Big(-\tfrac{1}{3} + \xi^2\Big)\phi^3 - \xi^2\phi^4 + \mathcal{O}(\xi^4)\,. \end{aligned}$$

Get a standard kinetic term plus a modified potential to first order in ξ^2.

Induction hypothesis: After some redefinitions, the Lagrangian takes the form

$$\tilde{L} = \; L_0 \; - \; \xi^2 \tilde{V}_2 - \cdots - \xi^{2k-2} \tilde{V}_{2k-2} + \xi^{2k} L'_{2k} + \mathcal{O}(\xi^{2k+2}) \, .$$

Want to show we can remove all the derivatives in L'_{2k} with a field redefinition

$$\phi \to \phi + \xi^{2k} \delta_{2k} \phi \, ,$$

$$\begin{aligned} \tilde{L} + \Delta\tilde{L} = \; & L_0 \; - \xi^2 \tilde{V}_2 - \cdots - \xi^{2k-2} \tilde{V}_{2k-2} \\ & + \xi^{2k} \left(L'_{2k} - (\delta_{2k}\phi) \left[(\partial_t^2 \phi + V'(\phi) \right] \right) + \mathcal{O}(\xi^{2k+2}), \end{aligned}$$

To eliminate the higher derivatives, we need that any term $X_{2k} \in L'_{2k}$ can be written in the form

$$X_{2k}(\phi) = (\partial_t^2 \phi) X_0(\phi) \to -V'(\phi) X_0(\phi) = -V'(\phi)(\partial_t^2 \phi) X_1(\phi) \to \cdots$$

The general term X_{2k} is of the form:

$$T = (\partial_t^{k_1}\phi)\ (\partial_t^{k_2}\phi)\cdots(\partial_t^{k_\ell}\phi)\ (\partial_t\phi)^r\ \phi^s\,.$$

$$r, s \geq 0\,, \quad 3 \leq k_1 \leq k_2 \leq \cdots \leq k_\ell$$

T is a term of *index* ℓ.

k_1 the *lowest order* of T.

Claim Integration by parts of a time derivative in the first factor, reduces the *lowest order*. When the lowest order becomes 2, the next redefinition reduces the index. In this way we get a term with just first derivatives and powers of the field.

For first derivatives use

$$(\partial_t\phi)^{2p}\phi^q \ \ \rightarrow \ \ \frac{2p-1}{1+q}(\partial_t\phi)^{2p-2}V'(\phi)\phi^{q+1}\,.$$

to lower them recursively. $\square$

$$L = -\tfrac{1}{2}\,\phi\partial_t^2\phi - \tilde{V}(\phi;\xi^2)\,.$$

$$\begin{aligned}\tilde{V}(\phi;\xi^2) = -\tfrac{1}{2}\phi^2 &+ \left[-\tfrac{1}{3} + \xi^2 - \tfrac{3}{2}\xi^4 + 2\xi^6 - \tfrac{9}{4}\xi^8 + \cdots\right]\phi^3 \\ &+ \left[\xi^2 - \tfrac{19}{3}\xi^4 + \tfrac{419}{18}\xi^6 - \tfrac{4595}{72}\xi^8 + \cdots\right]\phi^4 \\ &+ \left[-\tfrac{16}{3}\xi^4 + \tfrac{517}{9}\xi^6 - \tfrac{12331}{36}\xi^8 + \cdots\right]\phi^5 \\ &+ \left[\tfrac{118}{3}\xi^6 - \tfrac{9194}{15}\xi^8 + \cdots\right]\phi^6 \\ &+ \left[-\tfrac{15812}{45}\xi^8 + \cdots\right]\phi^7 + \mathcal{O}(\phi^8)\ .\end{aligned}$$

Original potential $V(\phi) = -\frac{1}{2}\phi^2 - \frac{1}{3}\phi^3$ has a critical point at $\phi_* = -1$ and $V(\phi_*) = -1/6$.

$\tilde{V}(\phi;\xi^2)$ has a critical point at $\phi_*(\xi)$ and $V(\phi_*(\xi);\xi^2) = -1/6$.

Why? Write the original Lagrangian as

$$L \;=\; -K(\phi,\partial\phi;\xi^2) - V(\phi)\,,$$

K includes all terms with time derivatives of the fields. The field redefinition that gives us the local theory is

$$\phi \to \phi + g(\phi;\xi^2) + h(\phi,\partial\phi;\xi^2)$$

This means that $\tilde{V}(\phi;\xi^2) = V(\phi + g(\phi;\xi^2))$, that is, $\tilde{V}$ is just a redefinition of V.

Quasi-symmetries and ambiguities of the potential

A field redefinition which changes the potential also would change the kinetic term.

Surprisingly, this is not true for a nonlocal theory. There are field redefinitions that change the potential while leaving the kinetic term *unchanged at linearized order*.

This is a quasi-symmetry.

$$L = -\tfrac{1}{2}\phi\partial_t^2\phi - V(\phi)\,.$$

The simplest quasi-symmetry is

$$\boxed{\hat{\delta}_1\phi = c_1(\dot{\phi}^2 + V(\phi))}$$

$$\begin{aligned}
\rightarrow \quad L + \hat{\delta}_1 L &= L - \hat{\delta}_1\phi(\ddot{\phi} + V'(\phi)) + \mathcal{O}((\delta\phi)^2) \\
&= L - c_1(\dot{\phi}^2 + V)(\ddot{\phi} + V') + \mathcal{O}((\delta\phi)^2)\,, \\
&= L - c_1(\dot{\phi}^2\ddot{\phi} + V\ddot{\phi} + \dot{\phi}^2 V' + VV') + \mathcal{O}((\delta\phi)^2)\,.
\end{aligned}$$

total derivatives: $\quad \dot{\phi}^2\ddot{\phi} = \tfrac{1}{3}\partial_t\dot{\phi}^3\,, \quad V\ddot{\phi} + \dot{\phi}^2 V' = \partial_t(V\dot{\phi})\,.$

$$\rightarrow \qquad L + \hat{\delta}_1 L = L - c_1 VV' + \mathcal{O}((\delta\phi)^2)\,,$$

With $V = -\frac{1}{2}\phi^2 - \frac{1}{3}\phi^3$ and $c_1 = \beta_1\xi^4$ we get

$$L + \hat{\delta}_1 L = L - \beta\xi^4\left(\tfrac{1}{2}\phi^3 + \tfrac{5}{6}\phi^4 + \tfrac{1}{3}\phi^5\right) + \mathcal{O}(\xi^8)\,,$$

The potential is irrevocably changed at order ξ^4.

Next quasi-symmetry is:

$$\hat{\delta}_2 \phi = \dot{\phi}^4 + 3\dot{\phi}^2 V + \tfrac{3}{2} V^2 \,.$$

The general quasi-symmetry is

$$\hat{\delta}_n \phi = \quad \dot{\phi}^{2n} + (2n-1)\dot{\phi}^{2n-2} V + \cdots\cdots + (2n-1)!! \, \tfrac{1}{n!} V^n \,.$$

Using quasi-symmetries the potential can be put in special form with finite polynomials in ξ^2 for the odd powers:

$$\begin{aligned}
\tilde{V}(\phi;\xi^2) = -\tfrac{1}{2}\,\phi^2 &+ \left[-\tfrac{1}{3} + \xi^2\right]\phi^3 \\
&+ \left[\xi^2 - \tfrac{23}{6}\,\xi^4 + \tfrac{112}{9}\,\xi^6 - \tfrac{400}{9}\,\xi^8 + \tfrac{5056}{45}\,\xi^{10} - \tfrac{30848}{135}\,\xi^{12} + \tfrac{372224}{945}\,\xi^{14} + \cdots\right]\phi^4 \\
&+ \left[-\tfrac{13}{3}\,\xi^4 + \tfrac{370}{9}\,\xi^6 - \tfrac{2356}{9}\,\xi^8\right]\phi^5 \\
&+ \left[\tfrac{97}{3}\,\xi^6 - \tfrac{7444}{15}\,\xi^8 + \tfrac{40016}{27}\,\xi^{10} - \tfrac{16951588}{2025}\,\xi^{12} + \tfrac{365040328}{4725}\,\xi^{14} + \cdots\right]\phi^6 \\
&+ \left[-\tfrac{13532}{45}\,\xi^8 + \tfrac{1645424}{15}\,\xi^{10} - \tfrac{246594764}{6075}\,\xi^{12} + \tfrac{18403444376}{42525}\,\xi^{14}\right]\phi^7 \\
&+ \left[\tfrac{1057238}{405}\,\xi^{10} - \tfrac{528895198}{8505}\,\xi^{12} + \tfrac{293278365536}{297675}\,\xi^{14} + \cdots\right]\phi^8 \\
&+ \left[-\tfrac{17612426}{567}\,\xi^{12} + \tfrac{1376189404}{1323}\,\xi^{14} + \cdots\right]\phi^9 \\
&+ \left[\tfrac{17745598574}{42525}\,\xi^{14} + \cdots\right]\phi^{10} + \mathcal{O}(\phi^{11}).
\end{aligned}$$

Redefinitions of the general nonlocal theory

This time we consider general spacetime dependent fields

$$L = \frac{1}{2}\,\phi\partial^2\phi + \frac{1}{2}\,\phi^2 + \frac{1}{3}\left(e^{\xi^2\partial^2}\phi\right)^3, \quad \partial^2 = \partial^\mu\partial_\mu\,.$$

Can we remove higher-order derivatives perturbatively while keeping manifest Lorentz covariance?

Up to and including $\mathcal{O}(\xi^6)$, YES!

At $\mathcal{O}(\xi^8)$, however, we encounter an obstruction!!

Since the variation of the zeroth-order Lagrangian is now

$$\delta L_0 = \delta\phi(\partial^2\phi - V'(\phi)) + \mathcal{O}(\delta\phi)^2)$$

elimination of higher-order derivatives from a term X requires a writing

$$X = (\partial^2\phi)X_0(\phi) \;\to\; V'(\phi)X_0(\phi) = (\partial^2\phi)X_1 \to \cdots$$

Remove higher-order time derivatives from obstruction terms by breaking the manifest Lorentz covariance.

Not all terms with first-order derivatives can be removed, but $\phi^n(\partial\phi)^2$ for $n \geq 1$ can!

Writing $L = L_0 + \xi^2 L_2 + \xi^4 L_4 + \xi^6 L_6 + \xi^8 L_8 \cdots$, after field redefinitions we find

$$
\begin{aligned}
L_0 &= \tfrac{1}{2}\,\phi\partial^2\phi + \tfrac{1}{2}\,\phi^2 + \tfrac{1}{3}\,\phi^3 \\
L_2 &= -\,\phi^3 - \phi^4 \\
L_4 &= \tfrac{3}{2}\phi^3 + \tfrac{19}{3}\phi^4 + \tfrac{16}{3}\phi^5 \\
L_6 &= -\left(\tfrac{3}{2}\phi^3 + \tfrac{178}{9}\phi^4 + \tfrac{472}{9}\phi^5 + \tfrac{112}{3}\phi^6\right) - \tfrac{8}{3}(\partial\phi)^4 \\
L_8 &= \tfrac{9}{8}\phi^3 + \tfrac{223}{6}\phi^4 + \tfrac{483}{2}\phi^5 + 485\phi^6 + \tfrac{2695}{9}\phi^7 \\
&\quad + \left(\tfrac{52}{3} + 96\phi\right)(\partial\phi)^4 - \tfrac{4}{3}\,(\partial\phi)^2\partial^2(\partial\phi)^2
\end{aligned}
$$

- Up to and including ξ^6 there are no complications. Note that the term $(\partial\phi)^4 = [(\partial\phi)^2]^2$ is not higher derivative, and cannot be removed.

- The term in red is an <u>obstruction</u>. It is a higher-derivative term that cannot be written as $(\partial^2\phi)X$, using integration by parts. This term originates from the variation of the action under $\delta_6\phi$.

- We can deal with the obstruction by removing just the higher-order **time derivatives**. We thus break manifest Lorentz invariance.

Performing the needed redefinitions to order ξ^8,

$$
\begin{aligned}
L(\phi;\xi^2) = -\tfrac{1}{2}\,(\partial\phi)^2 - \left(\tfrac{8}{3}\,\xi^6 - 20\xi^8\right)(\partial\phi)^4 + \tfrac{304}{3}\xi^8\phi(\partial\phi)^4 \\
- \tfrac{8}{3}\,\xi^8(\partial\phi)^2\Big((\vec{\nabla}^2\phi)^2 + (\nabla_i\nabla_j\phi)^2 - 2(\vec{\nabla}\dot{\phi})^2 + 2(\phi^2+\phi)\vec{\nabla}^2\phi\Big) \\
\vdots \qquad\qquad \vdots \qquad\qquad \vdots \\
- \tfrac{1}{2}\,\phi^2 - \tfrac{1}{3}\,e^{-3\xi^2}\,\phi^3 \\
- \left[\xi^2 - \tfrac{19}{3}\xi^4 + \tfrac{178}{9}\xi^6 - \tfrac{653}{18}\xi^8 + \cdots\right]\phi^4 \\
- \left[-\tfrac{16}{3}\xi^4 + \tfrac{472}{9}\xi^6 - \tfrac{4307}{18}\xi^8 + \cdots\right]\phi^5 \\
- \left[\tfrac{112}{3}\xi^6 - \tfrac{7247}{15}\xi^8 + \cdots\right]\phi^6 - \left[-\tfrac{13451}{45}\xi^8 + \cdots\right]\phi^7 + \mathcal{O}(\phi^8).
\end{aligned}
$$

Remarks:

— The cubic term is exact.

— This procedure can be carried to all orders

— The theory has an initial value problem, no higher time derivatives.

— A Hamiltonian can be written.

Light-cone formulation

Coordinates: $x^+ = \tau, x^-, \mathbf{x}_T$

Light-cone time derivatives only appear in the kinetic term! Linearly!

$$
\begin{aligned}
L &= \tfrac{1}{2}\,\phi\partial^2\phi + \tfrac{1}{2}\,\phi^2 + \tfrac{1}{3}\phi^3 + \mathcal{O}(\xi^2)\,, \\
&= \tfrac{1}{2}\,\phi\Big(-2\partial_-\partial_\tau + \nabla_T^2 + 1)\phi + \tfrac{1}{3}\phi^3 + \mathcal{O}(\xi^2)
\end{aligned}
$$

Redefine the theory covariantly until there are terms with derivatives that cannot be removed and thus require analysis.

The first such term appears at ξ^6:

$$
L_6 \;=\; -\Big(\tfrac{3}{2}\phi^3 + \tfrac{178}{9}\phi^4 + \tfrac{472}{9}\phi^5 + \tfrac{112}{3}\phi^6\Big) - \tfrac{8}{3}(\partial\phi)^4
$$

The $(\partial\phi)^4$ term is not allowed in the light-cone: it has τ derivatives in the interaction.

To redefine away the derivatives consider the

$$\begin{aligned}\widetilde{T}_6 &= \delta\phi(\partial^2\phi+\phi+\phi^2)-\tfrac{8}{3}(\partial\phi)^4\\ &= \delta\phi(-2\partial_-\partial_\tau\phi+\nabla_T^2\phi+\phi+\phi^2)-\tfrac{8}{3}(-2\partial_\tau\phi\partial_-\phi+(\vec{\nabla}_T\phi)^2)^2\\ &= \delta\phi(-2\partial_-\partial_\tau\phi+\nabla_T^2\phi+\phi+\phi^2)-\tfrac{32}{3}(\partial_\tau\phi\partial_-\phi)^2+\tfrac{32}{3}(\partial_\tau\phi)(\partial_-\phi)(\nabla_T\phi)^2-\tfrac{8}{3}(\nabla_T\phi)^4.\end{aligned}$$

In light-cone one assumes ∂_- is invertible, so to remove a τ derivative we schematically

$$(\partial_\tau\phi)X=\frac{1}{\partial_-}(\partial_-\partial_\tau\phi)X=(-2\partial_-\partial_\tau\phi)\,\frac{1}{2\partial_-}X\;\rightarrow\;-(\nabla_T^2\phi+\phi+\phi^2)\,\frac{1}{2\partial_-}X$$

So that

$$(\partial_\tau\phi)X\;\rightarrow\;\left[\frac{1}{2\partial_-}(\nabla_T^2\phi+\phi+\phi^2)\right]X$$

A light-cone formulation can be reached.

Rolling Tachyons

The rolling field admits a series solution

$$\phi = \sum_{n=1}^{\infty} b_n e^{nt} = -e^t + b_2 e^{2t} + b_3 e^{3t} + \mathcal{O}(e^{4t}).$$

$$b_n = \frac{1}{n^2-1} \sum_{p=1}^{n-1} b_p b_{n-p} e^{-2\xi^2(n^2-np+p^2)}$$

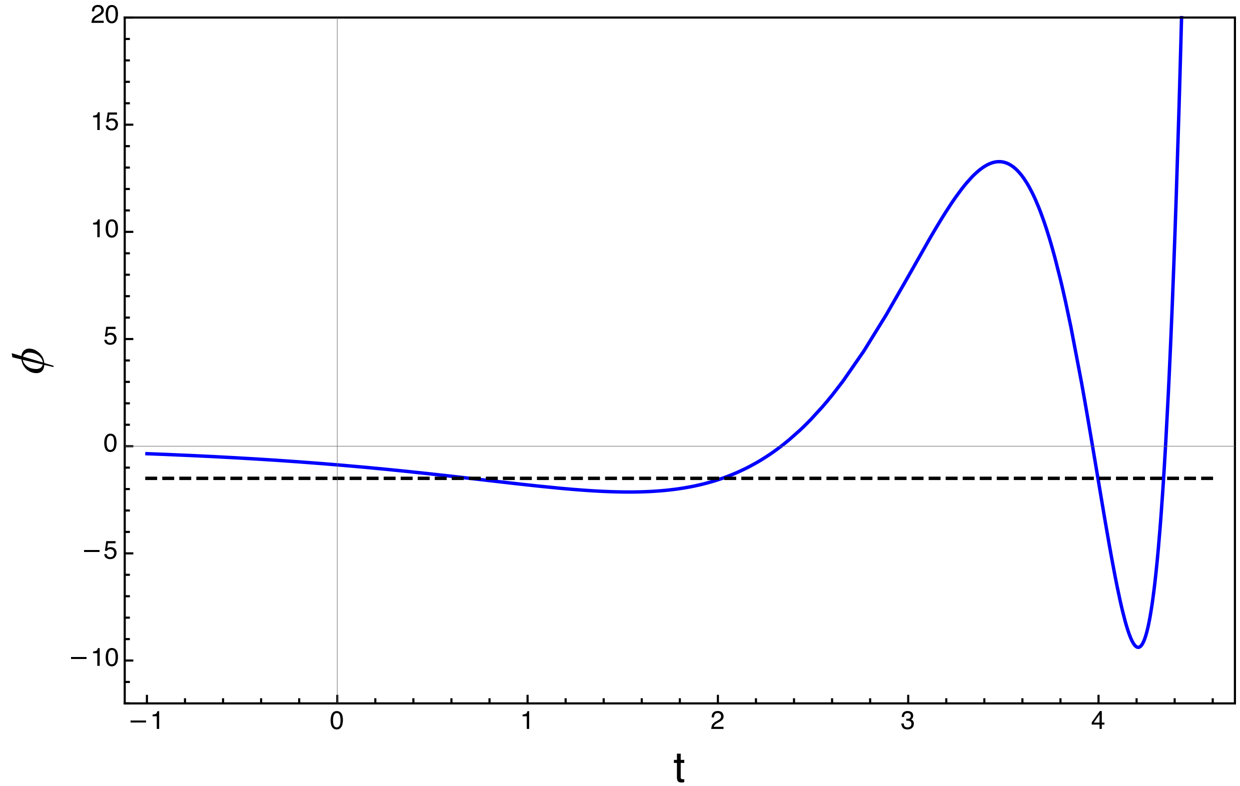
The first few coefficients are found to be

$$\begin{aligned} b_1 &= \; -1\,, \\ b_2 &= \; \tfrac{1}{3} e^{-6\xi^2}\,, \\ b_3 &= \; -\tfrac{1}{12} e^{-20\xi^2}\,, \\ b_4 &= \tfrac{1}{15}\left(\tfrac{1}{6} e^{-46\xi^2} + \tfrac{1}{9} e^{-36\xi^2}\right). \end{aligned}$$

Large n behavior of the coefficients

$$b_n \simeq (-1)^n 16 n^2 e^{-\beta n} e^{-3n^2\xi^2}, \quad \beta \simeq 2\,.$$

ensures convergence of the series expansion for all times.



The turning point at $\phi = -3/2$ is marked with black dashed line. Similar to the rolling solutions in p-adic strings, this also overshoots the turning points and oscillations get larger with time.

For $\xi = 0$ the series expansion above fails to converge after the tachyon field reaches the turning point.

But an exact solution exists that provides the analytic continuation:

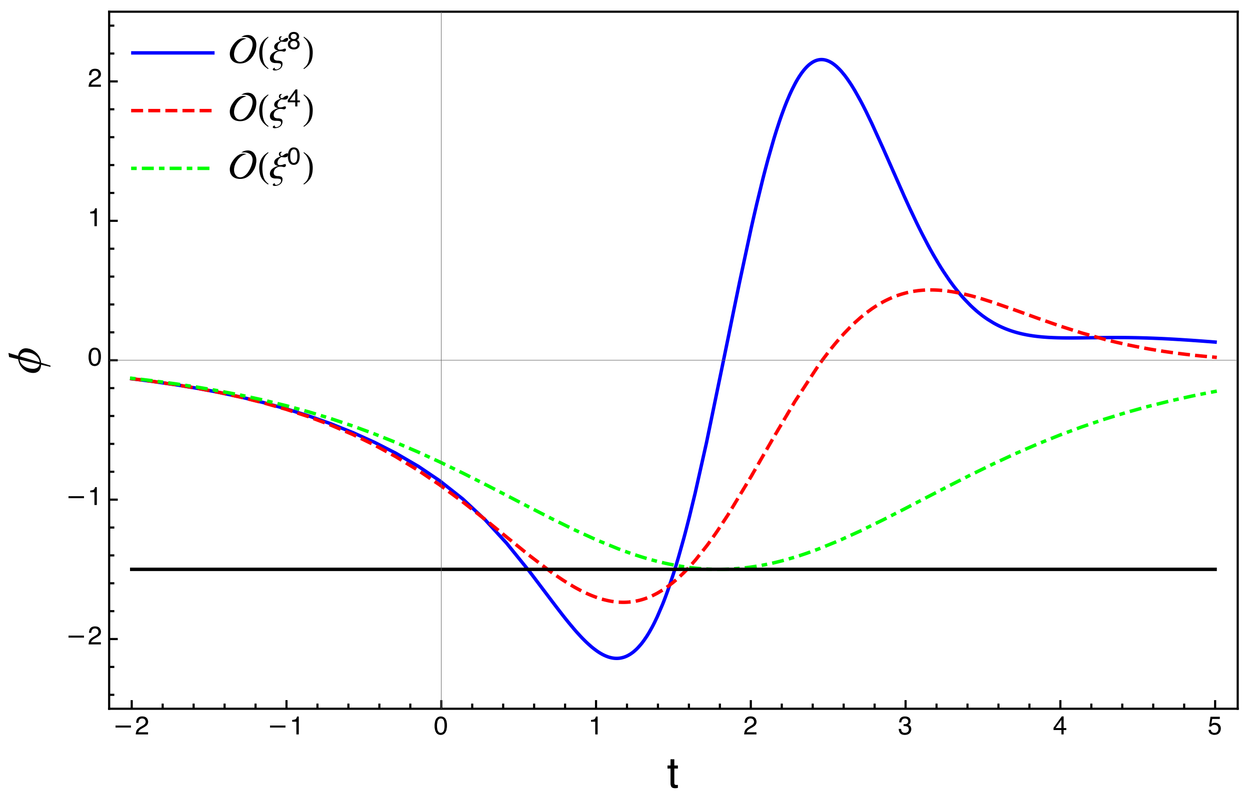
$$\phi_0(t) = -\frac{e^t}{(1+\frac{1}{6}e^t)^2}\,, \qquad -\infty < t < \infty\,.$$

Set up an expansion

$$\phi = \phi_0 + \xi^2\phi_2 + \xi^4\phi_4 + \mathcal{O}(\xi^6)\,,$$

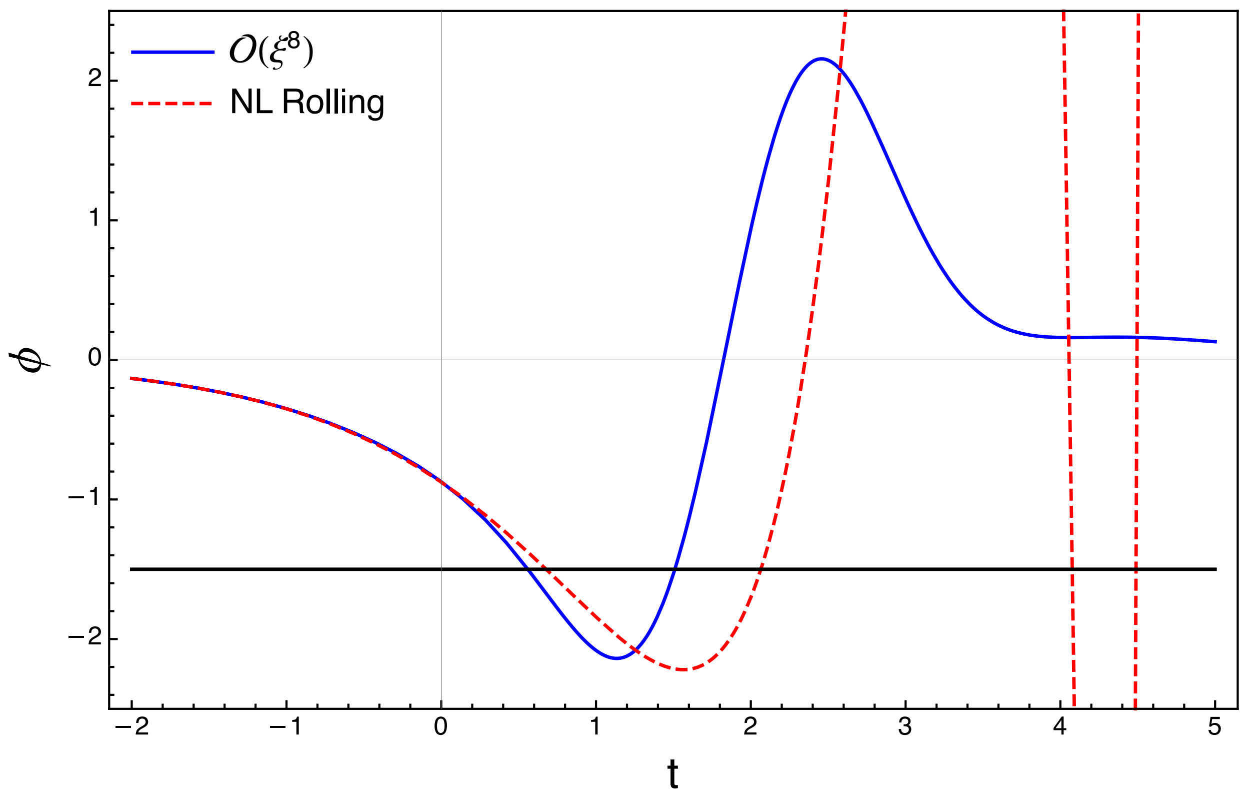
The rolling differential equations can be solved analytically at each order!

$$\phi = -\frac{36e^t}{(6+e^t)^2} + \frac{432e^{2t}\left(e^t - 6\right)}{(6+e^t)^4}\xi^2 - \frac{864e^{2t}\left(2e^{3t} - 129e^{2t} + 576e^t - 324\right)}{(6+e^t)^6}\xi^4 + \cdots$$



$\xi = 0.4$

Solutions in the local theory, and with the DE ξ^2 expansion. Note the overshooting of the turning point.

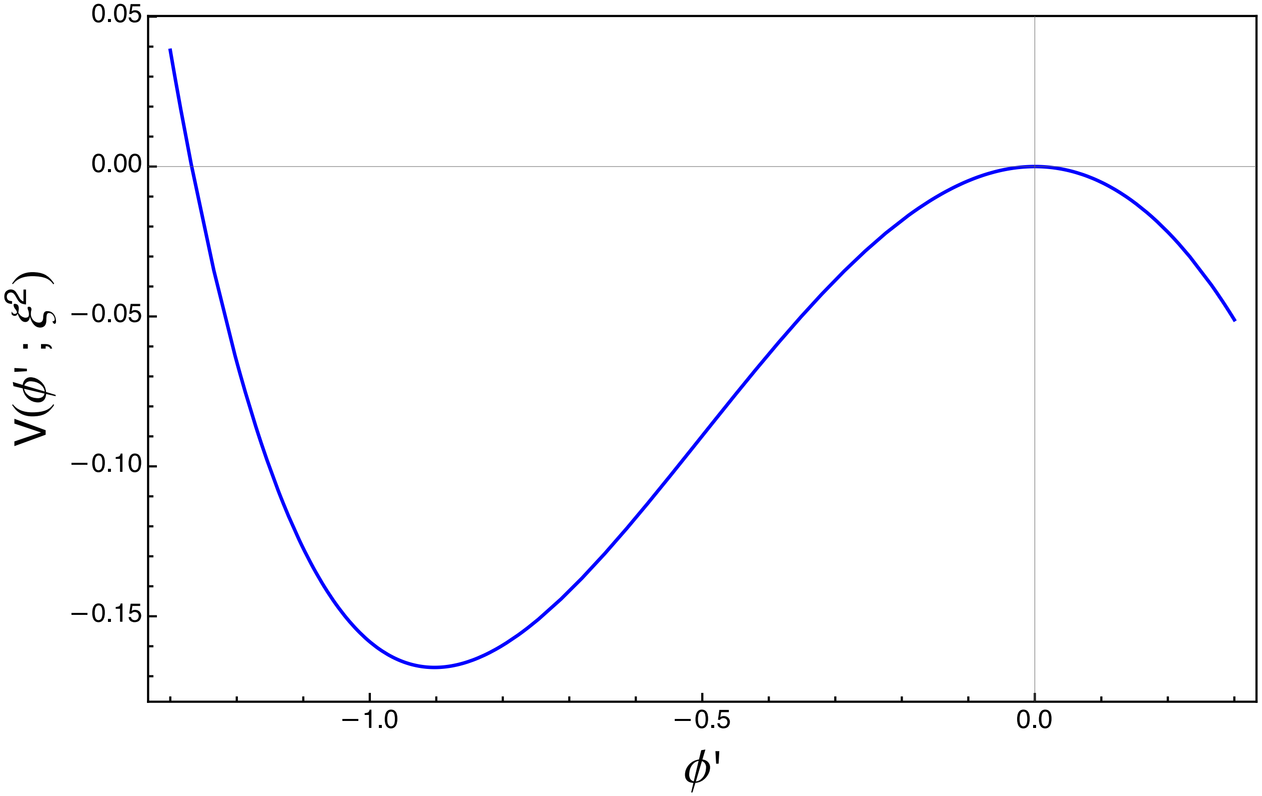


The DE solution (to order ξ^8) compared with the b_n series-solution ($n \leq 14$).

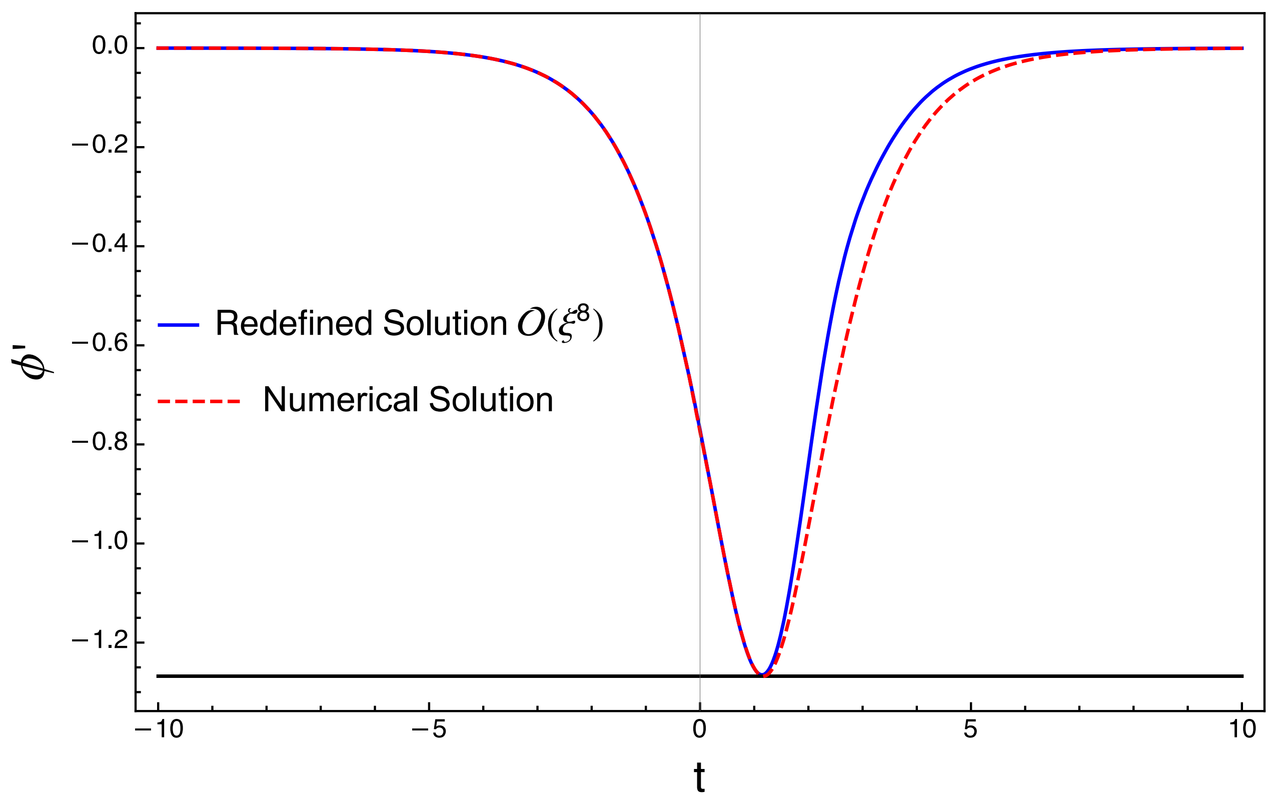
Better agreement would require a DE solution to order ξ^{100}

Insert the nonlocal theory rolling solution (from the DE-ξ^2) into the field redefinition to find the rolling of the redefined field.

Compare to the rolling of the canonical field on the potential $\tilde{V}(\phi;\xi^2)$.



Potential $\tilde{V}(\phi;\xi^2)$. Turning point at $\phi = -1.27$.

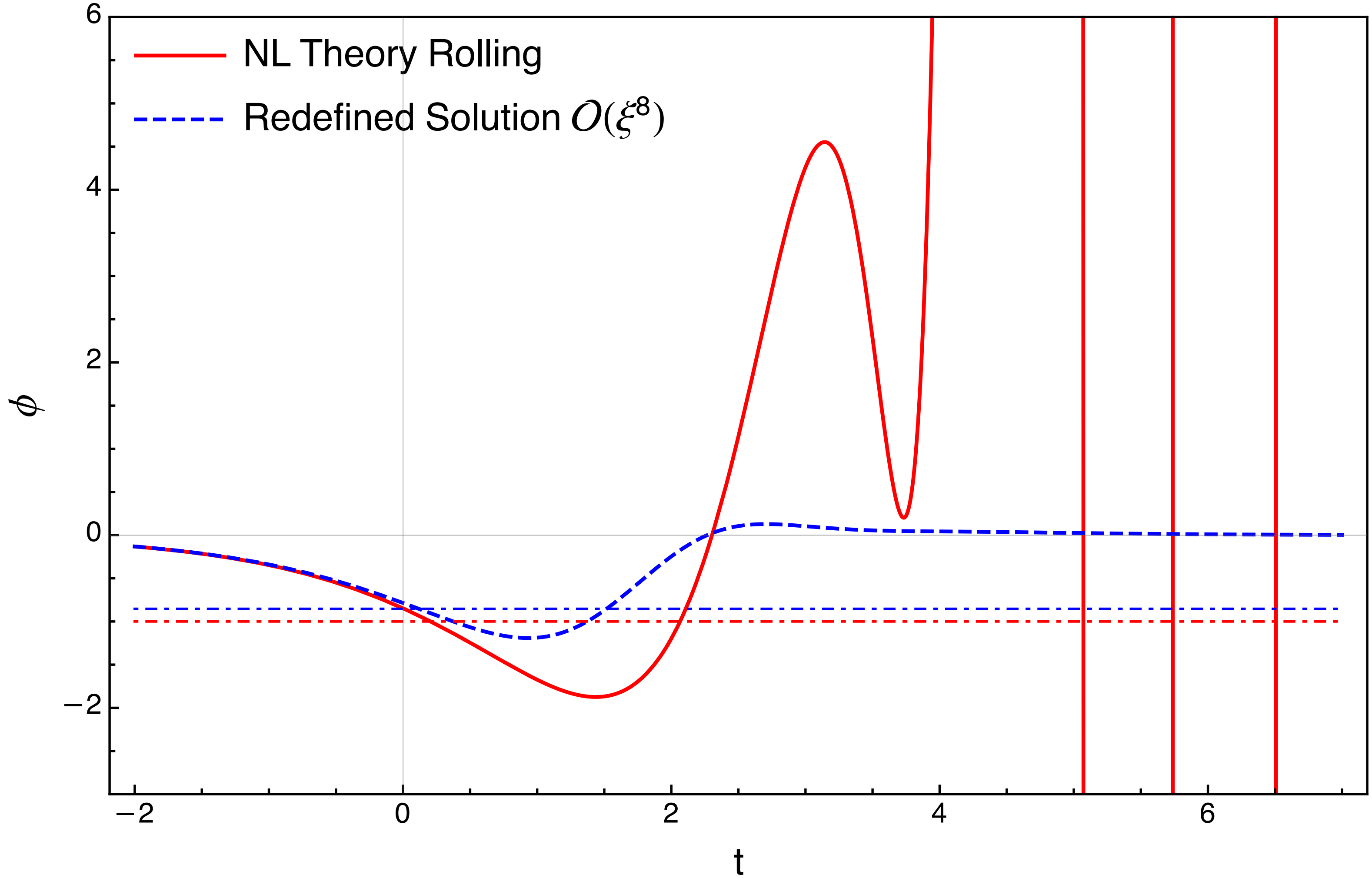


Solid blue line: Nonlocal theory DE solution inserted into the field redefinition.

Dashed red line: Canonical rolling in the potential $\tilde{V}(\phi; \xi^2)$.

Nice fact: the blue line does turns at the turning point of $\tilde{V}$.

The redefinition is doing the right thing, and mapping the NL theory solution into rolling in $\tilde{V}$.



Nonlocal theory rolling solution ($\xi = 0.35$)

Nonlocal DE solution, redefined.

Horizontal lines represent the tachyon vacuum vev (-1 for NL, and -0.8 for $\tilde{V}$).

Both rolling solutions cross the respective tachyon vacua nearly at the same time.

— The full rolling solution on the redefined potential goes from the unstable vacuum (D-brane) to the turning point (no-D-brane) and back at $t = \infty$ to the unstable vacuum (D-brane).

—It does look like the process of a brane decaying and re-forming again.

— This seems inconsistent with the nonlocal theory rolling solution, where the large time behavior seems inequivalent to the early time behavior.

—Is the NL solution really mapped back to the unstable vacuum at $t = \infty$. Our numerical work cannot yet do this!

Remarks and directions.

— Full locality can be achieved for the purely time-dependent situation

— An initial value problem can be formulated for general configurations at the cost of losing manifest Lorentz invariance.

— Moreover, our analysis suggests but does not demonstrate that the theory in question is causal.

—A closed form expression for the potential $V(\phi; \xi^2)$ relevant to the purely time-dependent theory?

—A deeper investigation of the causality of the original nonlocal theory is warranted. Bogoliubov causality condition?

— After all, if the redefined theory is causal and equivalent to the NL theory, then the latter must be causal.